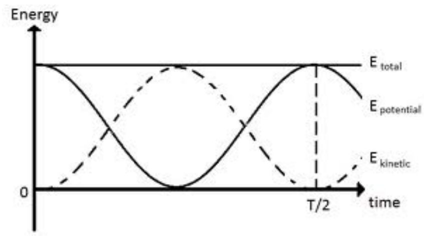
Chapter 10 - Simple Harmonic Motion



Where did you see the above graph in a lab before?

Periodic Motion - Motion that REPEATS the same way every time

Period = TIME for one cycle = T (s)

Types of Periodic Motion

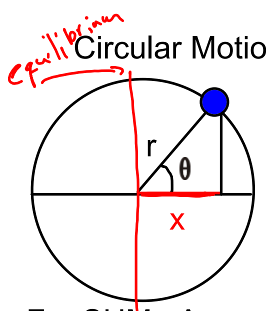
1) Circular

2) Pendulums

3) Springs

Also called **Simple Harmonic Motion** - repeats due to RESTORING force

Circular Motion and SHM



$x = r\cos\theta$

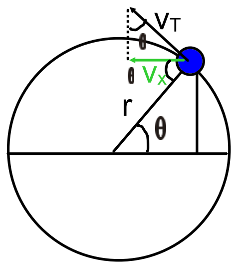
$x = r\cos\omega t$

$x = A\cos\omega t$

since cos max = 1

$x_{max} = A$

For SHM...A = amplitude = BIGGEST displacement



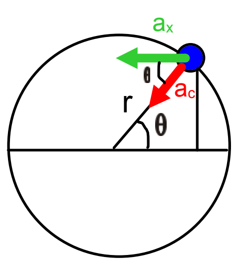
$v_x = -v_T \sin\theta$

$v_x = -r\omega \sin \omega t$

$$v_x = -A\omega \sin\omega t$$

since max sin = 1

$$v_{max} = -A\omega$$



$a_x = -a_c \cos\theta$

$a_x = -r\omega^2 \cos\omega t$

$$a_x = -A\omega^2 \cos\omega t$$

since cos max = 1

$$a_{max} = -A\omega^2$$

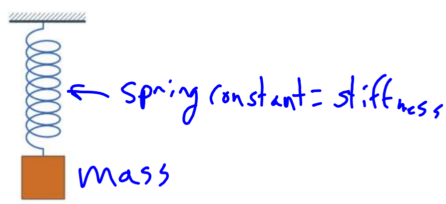
SHM for circular motion and calculus

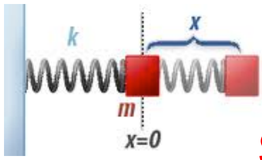
$x = A\cos\omega t$

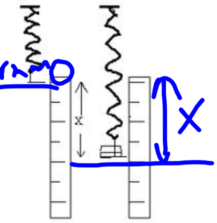
$x' = v = -A\omega \sin\omega t$

$x'' = a = -A\omega^2\cos\omega t$

What affects the period of a spring?







Spring constant (k) is INDEPENDENT of mass

x = distance from 0 (equilibrium point were $F_{net} = 0$)

Hooke's Law

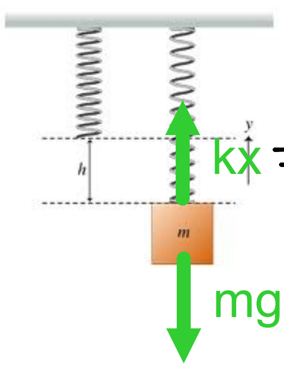
$F_s = -kx$

units = N/m

Spring const	k	N/m

Spring in Equilbrium

No motion!



$kx = F_s$

$kx = mg$

Mathematics of frequency of moving spring

$-Fx = ma$

$-k(A\cos\omega t) = m(-A\omega^2\cos\omega t)$

$\omega = \sqrt{k/m}$

= frequency in rad/s

f = frequency in rev/s....just divide by 2π

so $f = (1/2\pi)\sqrt{k/m}$

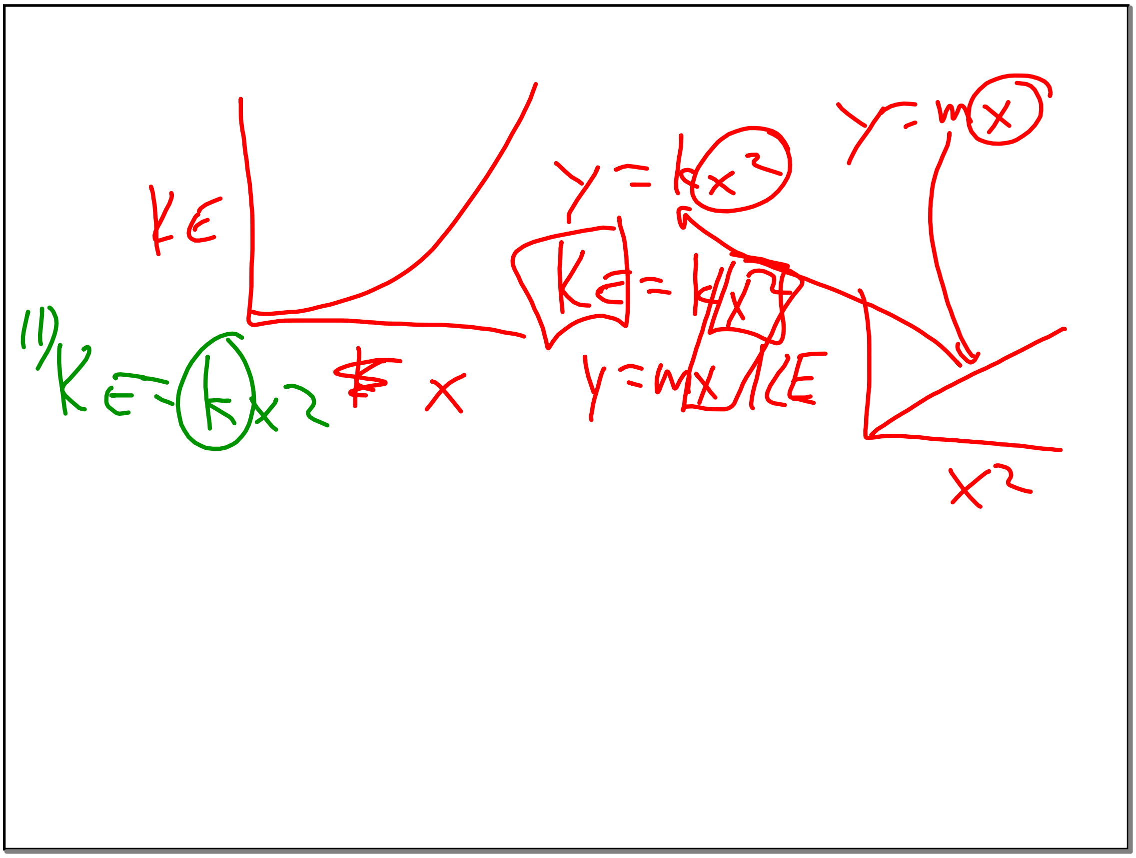
Only factors that matter are mass at the end and "Tightness" = spring constant = k

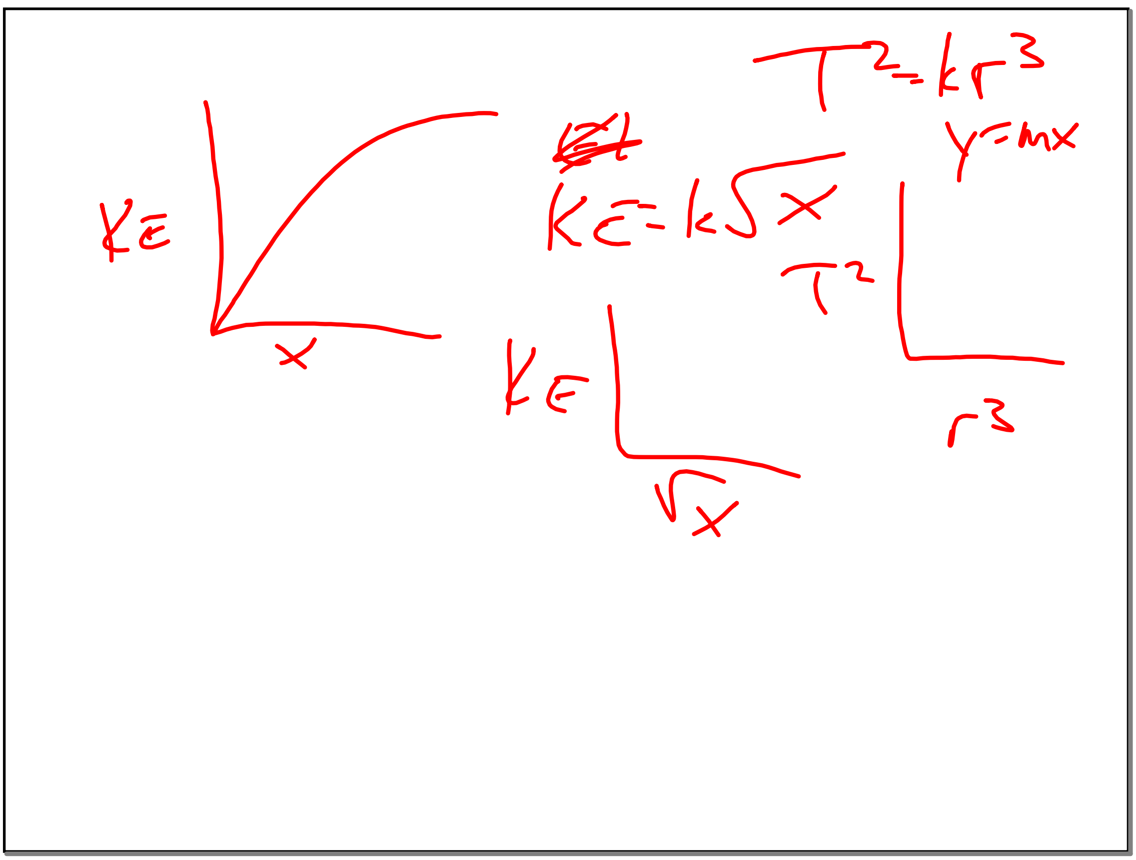
T is INVERSELY related to k

T is DIRECTLY related to m

$T_s = 2\pi\sqrt{m/k}$

note...T = 1/f so this is just f "inversed"

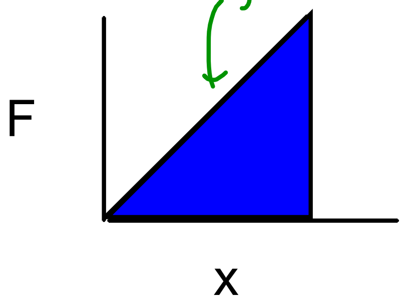

KE
y = kx²
y = mx
KE = kx²
y = mx
KE
x²
(1) KE = kx² x


T² = kr³
y = mx
KE
x
KE = k√x
T²
r³
KE
√x

Energy of Springs

$F = kx$

slope = k

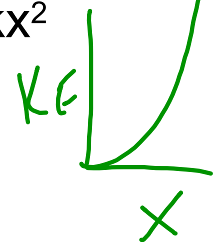


$W = \text{area} = 1/2Fx = KE = 1/2kx^2$

$1/2(kx)x = 1/2\, kx^2$

$U_s = 1/2kx^2$

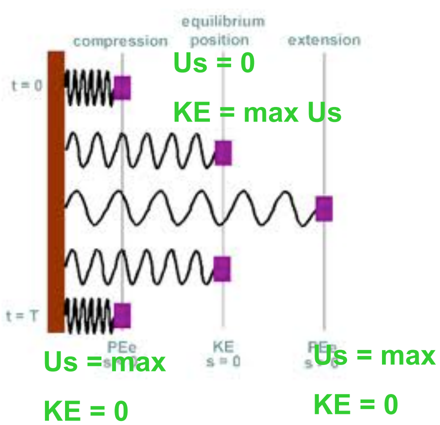
1/2k

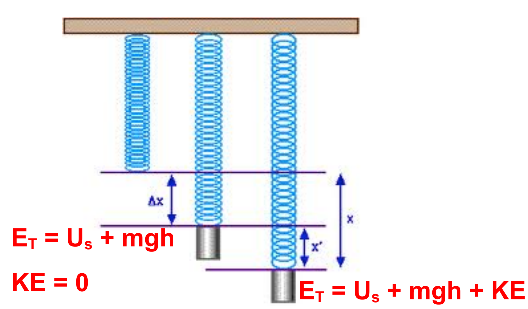


Conservation of Energy and Springs

$U_{go} + KE_o + U_{so} = U_{gf} + KE_f + U_{sf}$

$mgh_o + \frac{1}{2}mv_o^2 + 1/2kx_o^2 = mgh_f + 1/2mv_f^2 + 1/2kx_f^2$

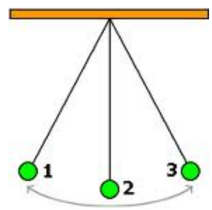




E_T = always the same!!

$= 1/2kA^2$

What affects the period/frequency of a pendulum?



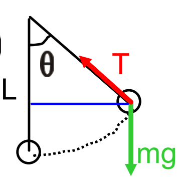
Length

gravity

$\tau = Fr = mgL\sin\theta$

For small angles

$\sin\theta = \theta$



so $\tau = mgL\theta$

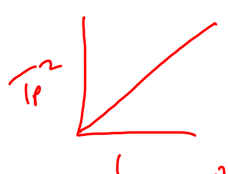
which is like $F = kx$

so k for pendulums = mgL, and angular m = I

$\omega = \sqrt{k/m} = \sqrt{mgL/I}$ but $I = mL^2$ so...

$= \sqrt{mgL/mL^2}$

$\omega = \sqrt{g/L}$ and $f = (1/2\pi)\sqrt{g/L}$ and $T_p = 2\pi\sqrt{L/g}$



$m = \frac{4\pi^2}{g}$

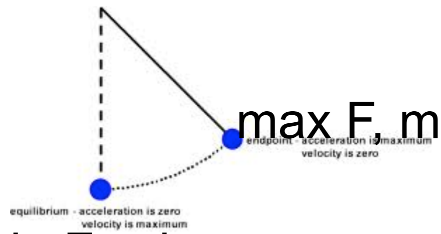
$T_p^2 = 4\pi^2\left(\frac{L}{g}\right)$

$T_p^2 = \left(\frac{4\pi^2}{g}\right)L$

Length and gravity are only variables that matter

T and L are DIRECTLY related

T and g are INVERSELY related



max F, max a

min F, min a =
equilibrium point =
Fnet = 0

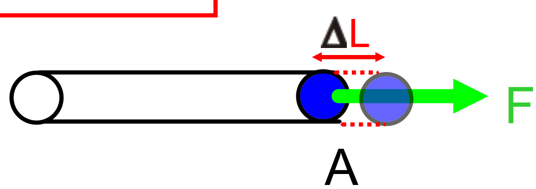
Elasticity - distortion of object due to applied force that disappears when force disappears

Linear Deformation

$F = Y(\Delta L/L_o)A$

$F = kx$

Y = Young's Modulus



F/A = stress

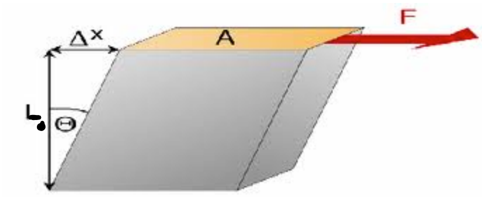
$\Delta L/L_o$ = strain

***note F is ⊥ A

Sheer Deformation

$F = S(\Delta x/Lo)A$

S = sheer modulus

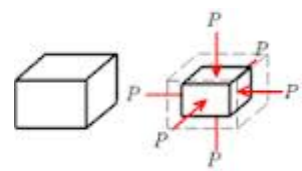


F/A = stress

$\Delta x/L_o$ = strain

***Note F and A are //

Volume Bulk Deformation



P = pressure   P = F/A = N/m^2 = Pascals = Pa

$P = - B(\Delta V/V_o)$

B = Bulk Modulus (N/m^2)

P is all around neither
// nor ⊥

Stress = P

Strain =$\Delta V/V_o$