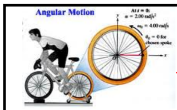


Chapter 8 - Rotational Kinematics

Units for rotational are radians or degrees or revolutions

360 deg = 2π rad = 1 revolution

Need to be able to convert

360 degrees
OR
2π radians

Time to learn Greek!

All rotational measurements have greek equivalents

x = θ theta θ ϕ

v = ω omega ω

a = α alpha α

Big 5 Kinematic EQ's

$\bar{\theta} = 1/2(\omega_o + \omega_f)t$

$\theta = 1/2\,\alpha t^2 + \omega_o t + \theta_o$

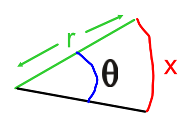
$\omega_f = \alpha t + \omega_o$

$\omega_f^2 = 2\alpha\theta + \omega_o^2$

$\bar{\omega} = \theta/t$

Converting between linear and angular

Just multiply by r



r θ = x

r w = v

r α = a

BUT MUST BE in **radians**

θ, ω, α

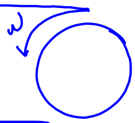
Let's go back to centripetal acceleration

tangential velocity

Remember... v_t

$\boxed{a_c = v^2/r}$ (what kind of v?)

ω



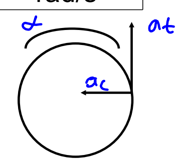
since $v = r\omega$

$a_c = (r\omega)^2/r = r\omega^2$ $\boxed{a_c = r\omega^2}$

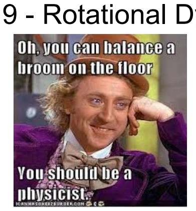
a_t	a_c	α
tangential	centripetal	angular
exists only if ω changes	exists ~~as~~ if moving in circle	exists only if ω changes
$= r\omega$	$= r\omega^2$	$= (\omega_f - \omega_o)/t$
m/s^2	m/s^2	rad/s^2

Total acceleration:

$(a_{total})^2 = a_t^2 + a_c^2$



Chapter 9 - Rotational Dynamics

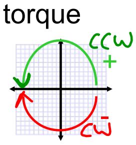


Torque = force that causes rotation = τ thao

> Must NOT be at pivot point

> Can be + (CCW) or - (CW)

> BIG lever arm = further from pp = BIG torque

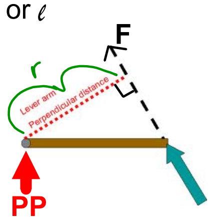


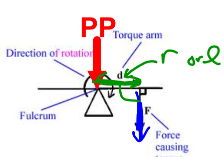
$\tau = Fr$

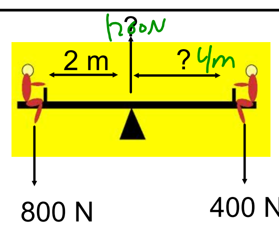
units = Nm

$r = \ell$

Lever arm = length of perpendicular line dropped from the axis to the line of force = r or ℓ





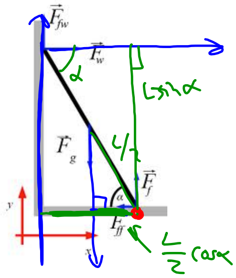


When doing torque problems it is like F_{net} problems. When in equilibrium...

$\Sigma F_y = 0$

$\Sigma F_x = 0$

$\Sigma \tau = 0$

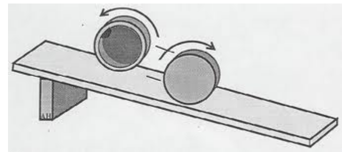


$\Sigma F_x = F_w - F_{ff} = 0$

$\Sigma F_y = F_{fw} + F_f - F_g = 0$

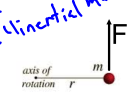
$\Sigma \tau = F_g\left(\frac{L}{2}\cos\alpha\right) - F_w(L\sin\alpha) - F_{fw}(L\cos\alpha) = 0$

But what happens when NOT in equilbrium?



Where mass is located relative to PP MATTERS! Not just amount of mass

= moment of inertia = "inertial mass" = I



$F = ma_t = mr\alpha$

$\tau = rF = r(ma) = \boxed{mr^2}\alpha$

so

$\boxed{\tau = Fr = I\alpha}$

$\tau = mr^2\alpha = I\alpha$

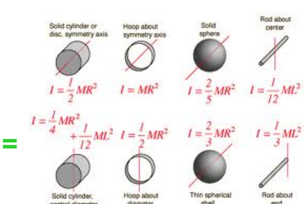
$I = I_d + 4mr^2$

$I = \sum_i m_i R_i^2$

units = kgm^2

Moment of Inertia is different for different objects...all listed on pg. 254...but should understand why some are bigger and smaller

Mass further from PP = MORE torque = more I



Greek equivalents in dynamics

F_{net} --> τ

m ---> I

Other equations translated into rotational greek

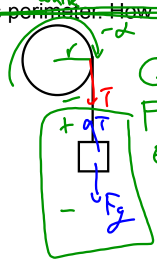
$KE_{rot} = 1/2(\mathbf{I}\boldsymbol{\omega}^2)$ $\boxed{\text{units = J}}$

$p = L = \mathbf{I}\boldsymbol{\omega}$ $\boxed{\text{units = kgm}^2\text{/s}}$

Ug = mgh...no different!

Sample Problem

A pulley with ~~an I = .8 kgm² and~~ r = 40 cm is rotating at 3 rad/s² ~~lifting~~ while a 2 kg mass by means of a string ~~wound on~~ drops ~~its perimeter. How far will it lift before stopping?~~



What is I for the pulley?

G: $r = .4m$, $\alpha = 3 rad/s^2$, $m = 2kg$

F: $I = ?$

E: $T - mg = ma = F_{net}$ $\quad r\alpha = a$

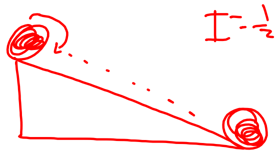
$$\boxed{\begin{array}{l} -Tr = I\alpha = \tau \\ T - mg = mr\alpha \end{array}}$$

Energy Conservation in rotational dynamics

$U_{go} + KE_{To} + KE_{roto} = U_{gf} + KE_{Tf} + KE_{rotf}$

Veritasium video on bullet and block with rotation

s://www.youtube.com/watch?v=N8HrMZB6_dU&list=PLSCMfVTUr-ZRZp4q3wCwMbGFsSzy2&index=7



$I = \frac{1}{2}mr^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$

$v = ?$

$r\omega = v$

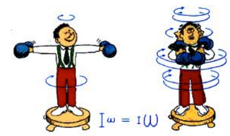
$\omega = v/r$

$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2$

$gh = \frac{3}{4}v^2$

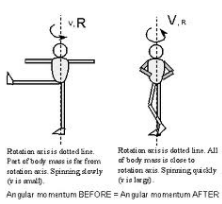
$v = \sqrt{4/3\, gh}$

Momentum Conservation in Rotational Dyamics



$I_o\omega_o = I_f\omega_f$

so smaller r = greater ω = faster!



$r\theta = x$

angular displacement	θ	rad
angular velocity	ω	rad/s
angular acceleration	α	rad/s²
torque	τ	Nm
moment of inertia	I	kgm²
angular momentum	L	kgm²/s

$m \cdot rad = m$

$I = mr^2$

$L = I\omega$